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Numerical Methods for Underwater Structural Acoustics Simulations

Howard Elman
Dianne P. O'Leary
Department of Computer Science
University of Maryland
College Park, MD 20742

Research Objectives

To develop and study solution algorithms for equations used to model reflection properties of underwater objects interacting with sonar signals.

Technical Approach and Accomplishments

The emphasis has been on development of algorithms for the numerical solution of the Helmholtz equation

$$-\Delta u - k^2 u = f, \tag{1}$$

the solution to which is used to determine the acoustic pressure associated with an underwater signal. Here we consider a three-dimensional box-shaped domain $\Omega = (a_1, b_1) \times (a_2, b_2) \times (a_3, b_3) \subset \Re^3$, with Sommerfeld-like boundary conditions

$$u_n - iku = 0 \tag{2}$$

on $\partial\Omega$.

Discretization of the problem (1)–(2) results in a linear system of equations

$$Au = f. \tag{3}$$

Since the problem is fully three-dimensional, any reasonable discretization will contain a large number of unknowns and require considerable storage. Direct methods based on Gaussian elimination with partial pivoting require a prohibitive amount of additional storage and thus have limited use. Multilevel methods suffer from the requirement that the coarsest spaces used must be fine enough to accurately represent the solution. In addition, the complex symmetric coefficient matrix A typically has eigenvalues with both positive and negative real parts. This can cause difficulties for iterative solution methods, and preconditioning of the matrix is essential in order to attain efficiency.

In this project, we solved the discrete Helmholtz equation using Krylov subspace iterative methods with a preconditioning methodology derived from fast

direct methods. The basic principle behind fast direct solvers is to apply an inexpensive transformation to break a problem into a number of lower-dimensional but independent problems. Many solvers use fast Fourier transforms (FFT's) to achieve separation of variables and then solve the resulting set of decoupled problems using sparse matrix methods. Fast direct methods are standard tools for solving the Poisson equation on regular domains with Dirichlet, Neumann or periodic boundary conditions; they can be adapted to other domains via capacitance matrix or embedding methods. They have been used for the three-dimensional Helmholtz equation with Dirichlet or Neumann boundary conditions on an irregular domain, and for the two-dimensional problem in polar coordinates with non-reflecting boundary conditions (derived from a Dirichlet-to-Neumann mapping). Here, we developed efficient solvers for problems with Sommerfeld-like boundary conditions on box-shaped domains. Combining our techniques with capacitance matrix methods would produce solvers for general geometries in Cartesian coordinates, including exterior problems.

Our accomplishments were threefold:

1. We approximated the discrete operator A with a matrix Q that can be treated with fast direct methods. For finite difference discretizations, we derived Q by defining and discretizing the differential operator in the same way as for A except that the boundary conditions on either two or four faces of Ω are replaced by more convenient ones (Dirichlet or Neumann). The resulting matrix Q differs from A by a (relatively) low-rank operator and can be used as a preconditioner for A, to accelerate the convergence of iterative solvers based on Krylov subspaces. We also developed variants of these ideas for finite element discretizations (on uniform grids), focusing on trilinear elements. Here, rather than explicitly modifying the boundary conditions to construct Q, we used the fact that the discrete operator A is close to a block Toeplitz matrix and replaced certain sub-blocks of A by Toeplitz approximations that are amenable to fast transforms. For both types of discretizations, we demonstrated empirically that Q meets the requirements for an effective preconditioner:

   - Applying the action of Q^{-1} to a vector is not too expensive. For our preconditioners, using Q^{-1} entails a set of FFT's together with solution of smaller dimensional problems.
   - Q greatly reduces the number of iterations needed by Krylov subspace methods to solve (3).

   In particular, we showed that for several choices of Q, the experimental convergence behavior of preconditioned restarted GMRES depends only mildly

on both the wave number k and the discretization mesh size. In addition, we demonstrated how the methods can be implemented on a parallel computer with high efficiency.

2. We then worked on analyzing this algorithm to confirm the empirical properties. We explicitly calculated the eigenvalues of the preconditioned operator. The main innovation is that the eigenvalues for two and three-dimensional domains can be calculated exactly by solving a set of one-dimensional eigenvalue problems. This permits analysis of quite large problems. For grids fine enough to resolve the solution for a given wave number, preconditioning using Neumann boundary conditions yields eigenvalues that are uniformly bounded, located in the first quadrant, and outside the unit circle. In contrast, Dirichlet boundary conditions yield eigenvalues that approach zero as the product of wave number with the mesh size is decreased. These eigenvalue properties yielded the first insight into the behavior of iterative methods such as GMRES applied to these preconditioned problems.

3. We have taken our algorithm for solving the discrete indefinite Helmholtz equation on a three-dimensional box-shaped domain with Sommerfeld-like boundary conditions and modified it to make it efficient for inhomogeneous media in which the speed of wave propagation is different on an interior domain. The preconditioners are of two types. The first is a preconditioner that we designed for a homogeneous medium. This works well if the volume of the interior domain is relatively small compared to the overall volume and if the speeds are close in value. The second preconditioner uses a direct solver on the interior domain in addition to the homogeneous preconditioner. We presented experimental results demonstrating that these algorithms display efficiency comparable to that we showed earlier for a homogeneous medium, making them quite useful for the problem of acoustic analysis with a submarine.

Relevance to the Navy

The equations of structural acoustics, of which (1) is one component (to be coupled with an equation of elasticity for a structure) model fluid-structure interaction in the presence of an acoustic wave. This model is used to identify the location and structural properties of underwater objects.

Table 1: Representative iteration counts for GMRES(20) with preconditioners that use two sets of one-dimensional transforms, for finite difference discretization with several wave numbers and grid sizes.

Q_{dd} (sine + 1D solves)

k \ m	20	40	60	80
1	14	15	13	12
5	12	13	14	14
10	13	14	15	15
20	20	25	21	21
30	36	30	29	28
40	34	53	47	45
50	46	76	69	69

Q_{nn} (cosine + 1D solves)

k \ m	20	40	60	80
1	5	4	3	3
5	10	9	8	8
10	19	17	16	15
20	50	41	38	35
30	87	76	77	69
40	133	96	106	95
50	264	142	174	165

Table 2: Representative CPU times for solution of finite element discretizations, with $k = 5$ and several grid sizes.

Q_d (sine + 2D solves)

Number of processors \ m	16	32	64
1	9.20	91.56	–
2	4.38	45.81	–
4	2.09	24.41	–
8	1.39	12.23	–
16	1.20	6.67	70.59

Q_{dd} (sine + 1D solves)

Number of processors \ m	16	32	64
1	3.41	24.82	–
2	1.71	12.70	–
4	.80	7.03	49.34
8	.65	3.71	25.10
16	.80	2.59	15.21

Representative Results

Our results are in the form of iteration counts and CPU times for solving the discrete Helmholtz equation (1). Representative statistics are given in Table 1 and 2. Table 1 shows iteration counts for solving the discrete problem on a three-dimensional $m \times m \times m$ grid for two variants of the preconditioner. Numbers above the jagged line correspond to accurate models (with at least ten grid points per wave). The results demonstrate the insensitivity of performance to mesh size $(1/m)$ as well as the relatively small dependence on wave number. Table 2 shows CPU times on an IBM SP-2 computer. These results demonstrate the scalability of the algorithms. We note that the largest problems that we have solved contain 729,000 degrees of freedom (on a $90 \times 90 \times 90$ grid) and required just two minutes of CPU time.

List of Publications / Reports / Presentations

1. Papers Published in Refereed Journals

   (a) Howard C. Elman and Dianne P. O'Leary "Efficient Iterative Solution of the Three-Dimensional Helmholtz Equation," *Journal of Computational Physics*, to appear, 1998.

2. Non-Refereed Publications and Published Technical Reports

   (a) Howard C. Elman and Dianne P. O'Leary, "Efficient Iterative Solution of the Three-Dimensional Helmholtz Equation," UMIACS-TR-97-63, University of Maryland, 1997.

   (b) Howard C. Elman and Dianne P. O'Leary, "Eigenanalysis of Some Preconditioned Helmholtz Problems," Computer Science Department Report CS-TR-3890, Institute for Advanced Computer Studies Report UMIACS-TR-98-22, University of Maryland, March 1998.

   (c) Howard C. Elman, Dianne P. O'Leary, and Ilya Zavorine, "Efficient Solution of the Three-Dimensional Helmholtz Equation with Varying Wave Number," to appear.

3. Invited Presentations

   (a) Howard C. Elman, "Efficient Iterative Solution of the Three-Dimensional Helmholtz Equation" Czech – U.S. Workshop on Iterative Methods and Parallel Computing, Milovy, Czech Republic, June 1997.

   (b) Dianne P. O'Leary, "The Helmholtz Equation: Capacitance Matrices, Domain Decomposition, and Beyond," Numerical Analysis Conference in honor of Olof Widlund, Courant Institute, New York University, January 1998.

4. Contributed Presentations

   (a) H. C. Elman, "Efficient Iterative Solution of the Three-Dimensional Helmholtz Equation" SIAM National Meeting, Minisymposium on Solution Methods for Navier-Stokes Equations, Finite Element Circus, New York University, April 1997.

   (b) H. C. Elman, "Efficient Iterative Solution of the Three-Dimensional Helmholtz Equation," Copper Mountain Conference on Iterative Methods, March 1998.

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Howard Elman and Dianne P. O'Leary

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University of Maryland
Department of Computer Science
College Park, MD 20742

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13. ABSTRACT *(Maximum 200 words)*

Algorithms were developed for the numerical solution of the Helmholtz problems with Sommerfeld-like boundary conditions. Discretizations by finite differences or finite elements can be effectively preconditioned using similar operators but separable boundary condition. We demonstrated empirically that the preconditioners are easy to apply and cause iterative methods such as GMRES to converge in a very modest number of iterations. Parallel implementation on the SP-2 computer enabled efficient solution of quite large problems.

We then provided the first analysis of such problems by noticing that the eigenvalues for two and three-dimensional domains can be calculated exactly by solving a set of one-dimensional eigenvalue problems. This observation permits analysis of quite large problems. Preconditioning using Neumann boundary conditions yields eigenvalues that are uniformly bounded, located in the first quadrant, and outside the unit circle. In contrast, Dirichlet boundary conditions yield eigenvalues that approach zero as the product of wave number with the mesh size is decreased. We have extended our algorithm to inhomogeneous media in which the speed of wave propagation is different on an interior domain. The algorithms display efficiency comparable to that for the homogeneous medium, making them quite useful for the problem of acoustic analysis with a submarine.

14. SUBJECT TERMS
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